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Perturbative Logarithms and Power Corrections in QCD Hadronic Functions. A Unifying Approach *

N. G. Stefanis †

Institut für Theoretische Physik II

Ruhr-Universität Bochum

D-44780 Bochum, Germany

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Abstract

I present a unifying scheme for hadronic functions that comprises logarithmic corrections due to gluon emission in perturbative QCD, as well as power-behaved corrections of nonperturbative origin. The latter are derived by demanding that perturbatively resummed partonic observables should be analytic in the whole Q^2-plane if they are to be related to physical observables measured in experiments. I also show phenomenological consequences of this approach. The focus is on the electromagnetic pion form factor to illustrate both effects, Sudakov logarithms and power corrections in leading order of $\Lambda_{\rm QCD}^2/Q^2$. The same approach applied to the inclusive Drell-Yan cross section enables us to perform an absolutely normalized calculation of the leading power correction in $b^2\Lambda_{\rm QCD}^2$ (b being the impact parameter), which after exponentiation, gives rise to a nonperturbative Sudakov-type contribution that provides enhancement rather than suppression, hence partly counteracting the perturbative Sudakov suppression.

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†Email: stefanis@tp2.ruhr-uni-bochum.de

1. INTRODUCTION

In recent years, effort in QCD has turned increasingly toward the problem of including resummation effects due to multiple soft gluon emission, both in perturbation theory, as well as in the nonperturbative regime. The first effect is related to Sudakov suppression [?], well-known from QED, whereas those in the nonperturbative regime manifest themselves as power-behaved corrections [?], which, after exponentiation, amount to a Sudakov-like form factor [?]. However, as it turns out [?] this contribution provides enhancement rather than suppression. The hope is that improving the perturbative and nonperturbative structure of the theory this way, it will be possible to get better agreement with the existing hadronic data in terms of both correct overall shape and also normalization. In these investigations the crucial organizing principle is QCD factorization, which provides a handle to separate the short-distance (hard) component of a reaction (controlled by the large mass scale in the process, Q) - that will be treated perturbatively - from its long-distance (soft) nonperturbative part, related to the nontrivial QCD vacuum structure (and field condensates).

In processes which involve the emission of virtual gluon quanta of low momentum, one must resum their contributions to all orders of the strong coupling constant. This gives rise to exponentially suppressing factors in b-space (where b is the impact parameter conjugate to the transverse momentum $Q_\perp$) of the reaction amplitude (or cross-section) of the Sudakov type with exponents containing double and single logarithms of the large mass scale of the process [?]. However, because of the Landau singularity of the running coupling at transverse distances $b \propto 1/\Lambda_{\rm QCD}$, an essential singularity appears in the Sudakov factor. Thus, one has to consider power corrections of $\mathcal{O}\left(b^2\Lambda_{\rm QCD}^2\right)$, which, though negligible for small b relative to logarithmic corrections $\propto \ln\left(b^2\Lambda_{\rm QCD}^2\right)$, may become important for larger values of the impact parameter.

In this talk, I will discuss a general methodology to treat (power) series in the running strong coupling in connection with gluon emission. To be more precise, I will address this issue in terms of two processes: one to which the OPE applies, viz. the pion electromagnetic form factor at leading perturbative order, and another, the Drell-Yan process, to which the OPE is not applicable. The first is a typical example of an exclusive process with registered intact hadrons in the initial and final states (for a recent review and references, see, e.g., [?]). Such processes provide a "window" to view the detailed structure of hadrons in terms of quarks and gluons at Fermi level (*Hadron Femptoscopy*). The Drell-Yan mechanism, on the other hand, has two identified hadrons in the initial state and a lepton pair (plus unspecified particles) in the final state, whose transverse momentum distribution is proportional to the large invariant mass of the materialized photon.

The goal in the second case will be to obtain not only the usual resummed (Sudakov) expression (which comprises logarithmic corrections due to soft-gluon radiation), but also to include the leading power correction as well, specifying, in particular, its concomitant coefficient. This becomes possible within a theoretical scheme, which models the IR behavior of the running coupling by demanding analyticity of physical observables (in the complex Q^2 plane) as a *whole* – as opposed to imposing analyticity of individual powers, i.e., order by order in perturbation theory –, while preserving renormalization-group invariance (references and additional information can be found in the recent surveys [?,?] and D.V. Shirkov, these proceedings). The underlying idea behind our method [?], is to demand that if hadronic observables, calculated at the partonic level, are to be compared with experimental data, they have to be analytic in the entire Q^2 plane. This "analytization" procedure encompasses Renormalization Group (RG) invariance (i.e., resummation of UV logarithms and correct UV asymptotics) and causality (which imposes a spectral representation). As we shall see below, *analytization* removes all unphysical singularities in the the IR region, rendering perturbatively calculated hadronic observables IR-renormalon free.

2. ANALYTIC FACTORIZATION SCHEME (AFS)

2.1 Perturbative Pion Form factor with Sudakov Corrections

Let us conduct our investigation by considering the space-like electromagnetic pion's form factor in the transverse (impact) configuration space:

$$F_\pi\left(Q^2\right) = \int_0^1 dxdy \int_{-\infty}^{\infty} \frac{d^2\mathbf{b}}{(4\pi)^2}\, \mathcal{P}_\pi^{\rm out}\left(y, b, P'; C_1, C_2, C_4\right) T_{\rm H}\left(x, y, b, Q; C_3, C_4\right) \\ \times \mathcal{P}_\pi^{\rm in}\left(x, b, P; C_1, C_2, C_4\right) + \ldots, \tag{1}$$

where the modified pion wave function is defined in terms of matrix elements, viz.,

$$\begin{aligned}\mathcal{P}_\pi\left(x, b, P, \mu\right) &= \int^{|\mathbf{k}_\perp|<\mu} d^2\mathbf{k}_\perp \mathrm{e}^{-\mathbf{i}\mathbf{k}_\perp\cdot\mathbf{b}_\perp} \tilde{\mathcal{P}}_\pi\left(\mathbf{x}, \mathbf{k}_\perp, \mathbf{P}\right) \\ &= \int \frac{dz^-}{2\pi} \mathrm{e}^{-ixP^+z^-} \left\langle 0 \left| \mathrm{T}\left(\bar{q}(0)\gamma^+\gamma_5 q\left(0, z^-, \mathbf{b}_\perp\right)\right)\right| \pi(P)\right\rangle_{A^+=0}\end{aligned} \tag{2}$$

with $P^+ = Q/\sqrt{2} = P^{-\prime}$, $Q^2 = -(P'-P)^2$, whereas the dependence on the renormalization scale μ on the RHS of (2) enters through the normalization scale of the current operator, evaluated on the light cone, and the dependence on the effective quark mass has not been displayed explicitly. In (2), $T_{\rm H}$ is the amplitude for a quark and an anti-quark to scatter via a series of hard-gluon exchanges with gluonic transverse momenta (alias inter-quark transverse distances) not neglected from the outset. In the above, the ellipsis indicates the non-factorizing soft part, as well as disregarded higher-order corrections. The scheme constants C_i emerge from the truncation of the perturbative series and would be absent if one was able to derive all-order expressions in the coupling constant. The scale C_1/b ($C_1 = C_3$) serves to separate perturbative from non-perturbative transverse distances (lower factorization scale of the Sudakov regime and *transverse* cutoff). The re-summation range in the Sudakov form factor is limited from above by the scale $C_2\xi Q$ (upper factorization scale of the Sudakov regime and *collinear* cutoff).[1] The arbitrary constant C_4 serves to define the renormalization scale $C_4 f(x,y)Q = \mu_{\rm R}$, which appears in the argument of the analytic running coupling $\alpha_{\rm s}^{\rm an}$ [**?**] (choice of renormalization prescription):

$$\begin{aligned}\bar{\alpha}_{\rm s}^{\rm an(1)}(Q^2) &\equiv \bar{\alpha}_{\rm s}^{\rm pert(1)}(Q^2) + \bar{\alpha}_{\rm s}^{\rm npert(1)}(Q^2) \\ &= \frac{4\pi}{\beta_0}\left[\frac{1}{\ln\left(Q^2/\Lambda^2\right)} + \frac{\Lambda^2}{\Lambda^2 - Q^2}\right],\end{aligned} \tag{3}$$

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[1] Note that $\sqrt{2}C_2 = C_2^{\rm CSS}$ [**?**].